Mathematical Modeling of Molten Slag Granulation Using a Spinning Disk Atomizer (SDA)

Hadi Purwanto and Tomohiro Akiyama
Center for Advanced Research of Energy Conversion Materials, Hokkaido University
Kita 13 Nishi 8, Kita-ku, Sapporo 060-8628 Japan
Email: hadi@eng.hokudai.ac.jp

Abstract

To optimize granulation process, a mathematical model of a spinning disk atomizer (SDA) was developed to produce particle from high-temperature molten slag. The model comprises three parts: 1) *fluid flow model* of molten slag on the spinning disk, 2) *physical model* of ligament formation of slag, and 3) *heat transfer model* of slag drops dispersed from the ligament. First, a 2-D fluid flow model was developed to evaluate the film thickness of slag layer and was calculated using the scalar equation method. The number and diameter of the ligaments formed were evaluated using the physical model. Finally, the heat transfer model was employed to evaluate the quenching rate and temperature distribution within the drop. The developed model was verified by comparing the calculated result with the experimental data of drop diameter, both of which were found to be in reasonable good agreement. The advantage of this model is that it can also be used to predict the cooling rate and temperature distribution within the particle, which controls the slag properties.

KEY WORDS: molten slag, mathematical model, spinning disk atomizer, dry granulation

1. Introduction

Spinning disk atomizer (SDA) technology is well-known and is mainly applied in the chemical, food, and metallurgical industries. With regard to the mechanism of slag granulation, several literatures have proposed theoretical formulas to evaluate the particle diameter of the drops granulated from the rotary atomizer[1-5]. Fraser was the first to propose an empirical formula to predict the diameter of the drops obtained from the rotary disk experiment, using oil. Other researchers have proposed equations for different mediums such as water or liquids with a low viscosity. All these empirical formulas were largely based on one aspect of the operating parameter. In other words, they were derived from a conventional, dimensionless analysis. However, the mechanism of granulation that depends on the device design and the physical properties of the molten slag with very high viscosity is obviously different from that reported in previous studies.

Despite its engineering significance, the phenomena of the slag granulation using SDA and the heat transfer behavior in the particle have not yet been theoretically elucidated. Therefore, the objective of this study is to develop a comprehensive model using SDA through experimental observation of the slag granulation process.

That is, the mechanism of slag granulation was first observed experimentally using a high-speed video camera and was then used as the basis of this model, in which the influence of the rotating speed of the disk on the diameter of slag was mainly examined for validation. In this model, an unsteady state heat transfer analysis of a single slag particle is also carried out to predict its cooling rate and the temperature distribution within the particle during the cooling process. This will provide important information on whether the slag is vitrified or not. The granulated slag, produced by quenching the molten slag, has properties of glass and can be used directly as a raw material in cement production.

2. Model Overview

The model is developed based on the careful observation of the granulation phenomena during the experiment. Figure 1 shows a schematic diagram of the experimental apparatus. In the experiments, amount of molten slag was poured onto the spinning disk that was rotated by a motor at the bottom. The flow of slag followed the gravity force before contact with the disk surface and then sprayed out to become granulate. The experiment was observed using a high-speed video camera. The granulated slag was collected on a particle collecting sheet. Figure 2 shows the photographs of the molten slag flow on the SDA disk. These images reveal that the granulation process of the molten slag started out as a thin film flowing from the discharged slag on the center of the SDA disk during rotation. Then, the film

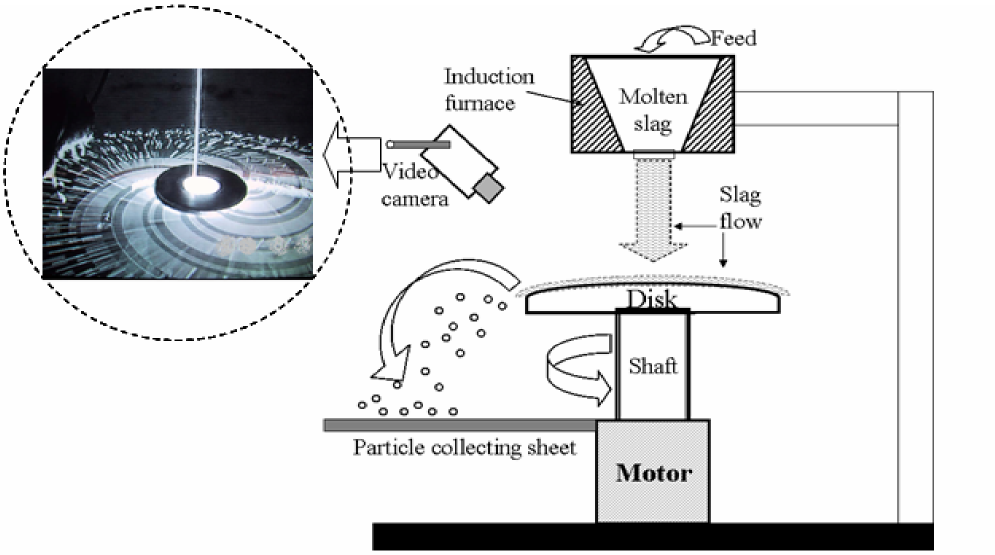


Fig.1 Schematic diagram experimental apparatus.

sprayed out of the disk due to centrifugal force to form a number of ligaments. Finally, the ligaments broke off to become granulates and were cooled while falling. Therefore, a comprehensive granulation model was constructed according to these three modes - thin film, ligament, and drop formation.

Fluid flow model of a thin film of slag on the rotating disk

The phenomena on the disk were assumed as a two dimensional coordinates system of an axial symmetrical cylinder. To obtain the numerical solution, a non-uniform computational mesh was used in finite differential equations based on SEM with appropriate initial and boundary conditions. A 100 × 70 × 10 grid was constructed for the numerical analysis. A fine grid was made at above the rotating disk, in which the molten slag flows from the center to the disk corner, and its behavior was analyzed.

The partial differential equations used to describe the liquid flow are the momentum and continuity equations. The variables to be computed are the volume fraction, slag flow rate, viscosity, rotating speed, and cup diameter. This model is developed based on the kinetic theory, at constant properties and under transient conditions, as shown by the following equations:

$$\rho\left(\frac{\partial u}{\partial t}+\frac{\partial u^2}{\partial x}+\frac{\partial uv}{\partial y}\right)=u\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{\partial P}{\partial x}+S_x \quad (1)$$

$$\rho\left(\frac{\partial v}{\partial t}+\frac{\partial uv}{\partial x}+\frac{\partial v^2}{\partial y}\right)=u\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\partial P}{\partial y}+S_y \quad (2)$$

Equations 1 and 2 represent the momentum or motion in x, y directions and $S_{x,y}$ is the momentum source. The above equations for the conservation of momentum should be supplemented with the continuity equation for incompressible flow as given below:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (3)$$

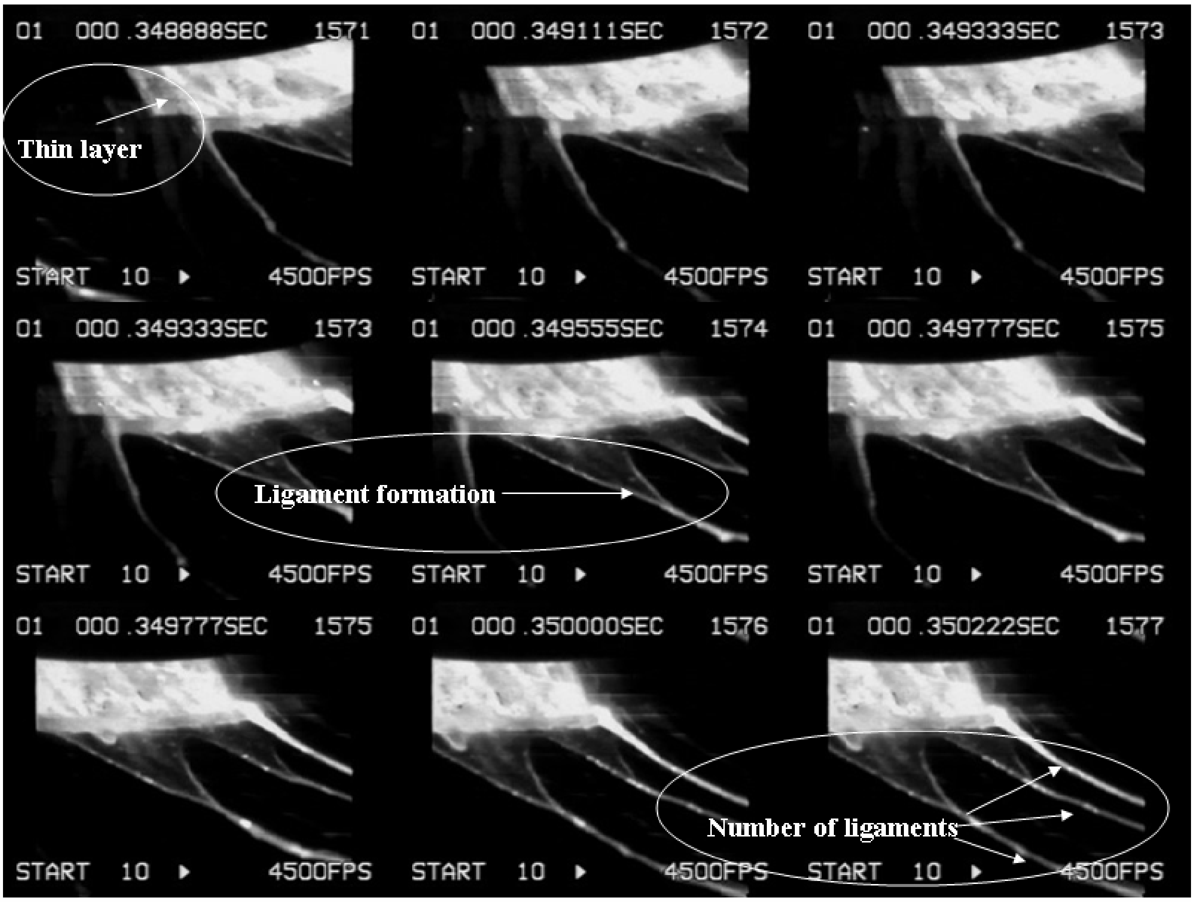


Fig.2 Photographs of ligament formation.

Physical model of ligament formation

A simple physical model based on a material balance was used for evaluating the number and diameter of the ligaments formed. That is, we assumed that all the slag reformed from the rotating disk changed into several ligaments with the same diameter. The number of the ligaments around the disk, *K*, was estimated using equations 5 to 7 under the operation conditions; additionally, the physical properties of the liquid slag and then the film thickness were also calculated. Accordingly, by substituting Eq. 8 in 9 we can calculate the nozzle diameter, D_e. Finally, the drop diameter from the liquid pillar was calculated by equation 8 using the predicted nozzle diameter.

$$We = \frac{\rho\omega^2 R^2}{\sigma_s} \tag{5}$$

$$Z_R = \frac{\mu}{\sqrt{\rho R \sigma_s}} \tag{6}$$

$$K = 0.31 We^{0.44} Z_R{}^{-0.36} \tag{7}$$

$$D_e = \phi\sqrt{\frac{4h(2\pi R/K)}{\pi}} \tag{8}$$

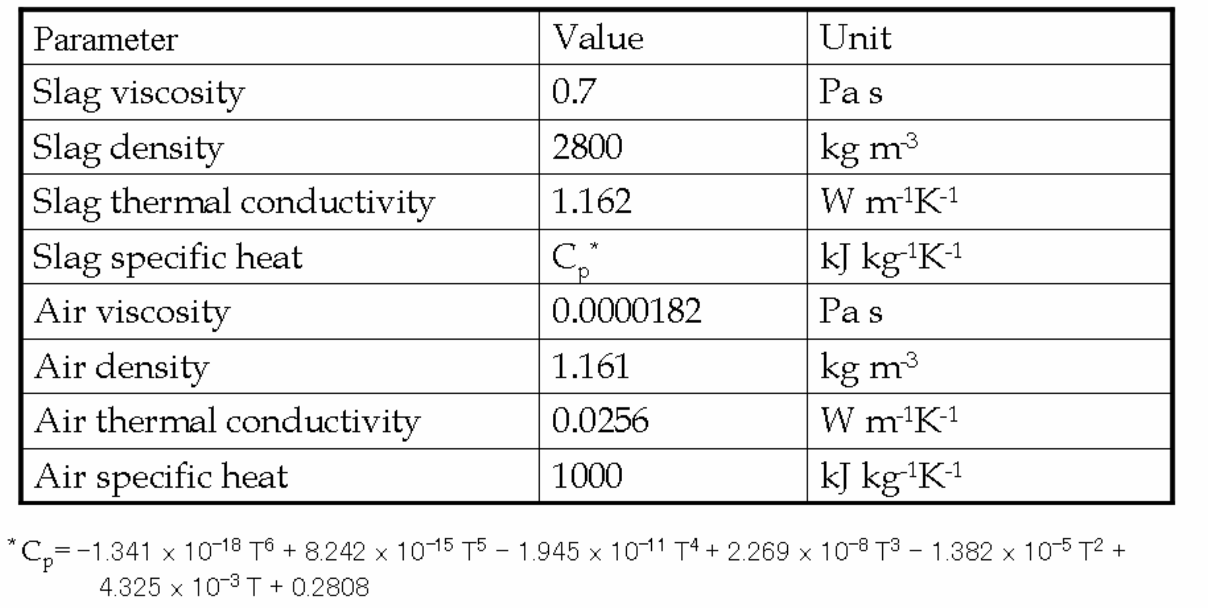
$$D_p = 1.89 D_e\left[1 + \frac{3\mu}{\sqrt{\sigma_s \rho D_e}}\right] \tag{9}$$

where, D_p is the diameter of particle drop. The viscosity of the slag was evaluated according to the method proposed by Iida et al.[10].

Heat transfer model of slag drop

Heat transfer analysis of a single particle was conducted according to the transient heat transfer method after the molten slag completely granulated [8-9]. Three heat transfer mechanisms are taken into consideration according to the transport phenomena analysis of convection between the particle surface and air, radiation between wall and particle, and conduction inside the particle. The fundamental equation used for the heat transfer analysis of a single particle is given as below:

Table 1. Properties used for the numerical computation



Parameter	Value	Unit
Slag viscosity	0.7	Pa s
Slag density	2800	kg m^{-3}
Slag thermal conductivity	1.162	W $m^{-1}K^{-1}$
Slag specific heat	C_p^*	kJ $kg^{-1}K^{-1}$
Air viscosity	0.0000182	Pa s
Air density	1.161	kg m^{-3}
Air thermal conductivity	0.0256	W $m^{-1}K^{-1}$
Air specific heat	1000	kJ $kg^{-1}K^{-1}$

$^* C_p = -1.341 \times 10^{-18} T^6 + 8.242 \times 10^{-15} T^5 - 1.945 \times 10^{-11} T^4 + 2.269 \times 10^{-8} T^3 - 1.382 \times 10^{-5} T^2 + 4.325 \times 10^{-3} T + 0.2808$

$$\rho C_p \frac{\partial T_s}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}(kr^2 \frac{\partial T_s}{\partial r}) + Q \qquad (10)$$

The governing equations for the analysis follow the assumptions:

The slag particle is spherical, and heat conduction occurs concentrically.

The volume change of the slag is negligible during phase transformation.

The temperature inside the slag is uniform, as an initial condition.

Boundary condition of particle:

r = R;

$$k\frac{\partial T_s}{\partial r} = h_p(T_s - T_g) + \varepsilon\sigma(T_s^4 - T_w^4) \qquad (11)$$

r = 0;

$$k\frac{\partial T_s}{\partial r} = 0 \qquad (12)$$

Initial condition:

$$T = T_0 \quad \text{at } t = 0 \qquad (13)$$

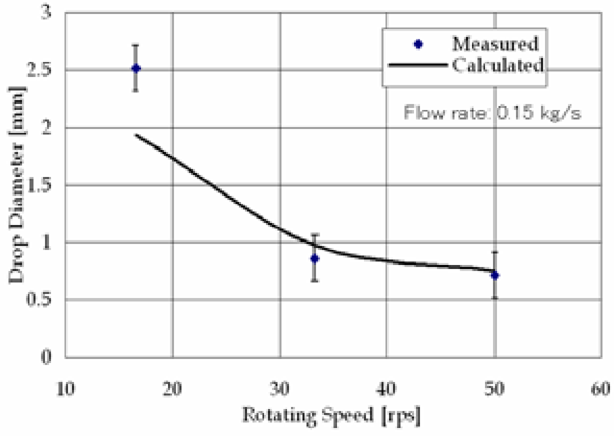


Fig.3 Comparison between the measured and calculated drop diameters.

This model then is solved by using the scalar equation method (SEM) [6-7]. In this method, density and viscosity are deduced from the values of variable Φ, as shown in Eq. 4 below:

$$\frac{\partial \Phi}{\partial t} + \frac{\partial \Phi u}{\partial x} + \frac{\partial \Phi v}{\partial y} = 0 \qquad (4)$$

where, Φ is a conserved scalar that has initial values of zero in one fluid (air) and unity in the other fluid (slag).

3. Results and Discussion

Verifying the model with experimental data

To verify the model, the calculation was demonstrated to predict the particle diameter of the granulated slag. The properties used for numerical computation are listed in Table 1. Figure 3 shows the slag particle diameter calculated based on the experimental conditions, using the above-mentioned method as a function of rotating speed. The solid line represents the calculated result from the developed model. The figure clearly indicates that the diameter of the granulated slag decrease with increasing in rotating speed, especially at an initial over 20 rps. The plots show that the data calculated by the model matches well with the experimental data[11], except when the rotating speed is less than 16.6 rps.

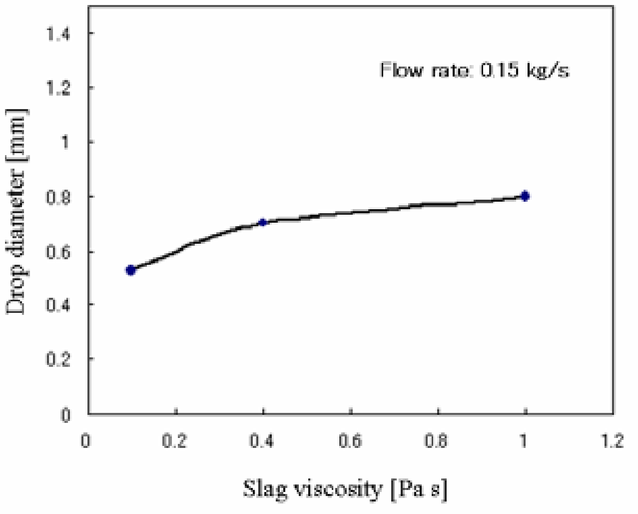


Fig.4 Effect of slag viscosity on drop diameter.

Regarding the experimental data, the deviation of the particle diameter at a lower rotating speed may be due to the fact that the shape of all the particles collected was not spherical and the distribution of particle size was very wide. A very few amount of fine particles are collected, however, large and heavy particles were dominantly produced under this condition. It may cause the deviation between measured data and the calculated of drop diameter using the model. In contrast, the granulated slag drops obtained from experiment at 50 rps are uniformly spherical with a diameter of less than 1 mm. Furthermore, calculations were conducted with difference of slag viscosity. The results are shown in Fig.4. The plots show that low viscosity of slag (0.1 Pa.s) resulted in particle with small diameter. Decreasing the viscosity of slag leads to produce fine granulated slag.

Temperature distribution of a single particle

Heat transfer analyses of the granulated slag during cooling process, were carried out for a single particle. In this analysis, the temperature of the molten slag on the disk was assumed to be constant. Figure 5 shows the changes in the calculated temperature of a single particle with diameters of 0.7 x 10^{-2} m. The results show that there is no temperature difference between the center and at the surface of particle. Temperature distribution in this particle was homogeneous. The temperature dropped rapidly in the few seconds before reached at constant condition. Experimentally, the time required for the particle to drop onto the collector surface was approximately 0.5 s. The time required for cooling the slag was in the range that for dropping. Therefore, the particle was cooled before dropping on the ground. It verifies that rapid cooling converts the molten slag into glassy and transparent particles. The high cooling rate resulted in high glassy property of granulated slag. In the near future, the model could be used to optimize the granulation process using SDA by conducting numerical experiments, and to control the properties of the granulate slag by determining the cooling rate based on particle diameter.

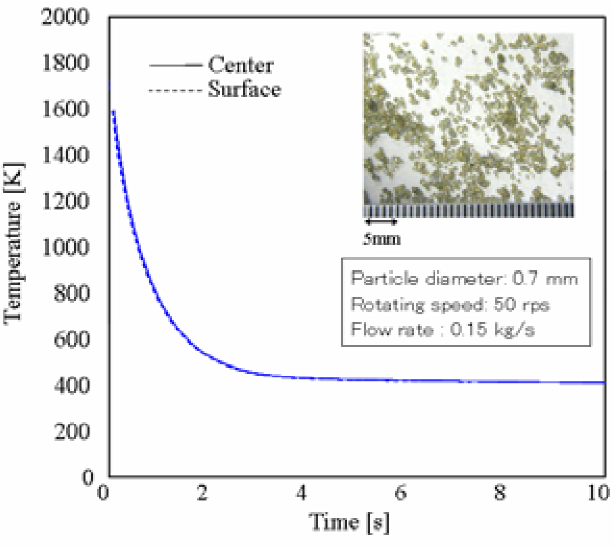


Fig.5 Calculated temperature distribution of a single particle during the cooling process.

4. Conclusions

The mathematical model developed in this works is capable to predict the drop diameter of slag produced by SDA. The model can be used for optimizing the granulation process using SDA from the viewpoints of drop size, glassification property of the product, and so on. A comparison the calculated result with the experimental data shows the validity of the model for predicting the drop diameter. The agreement is reasonable good. The advantage of this model is that it can also be used to predict the cooling rate and temperature distribution within the particle, which controls the slag properties.

Acknowledgements

A part of this research was supported by the Japan Society for Promotion of Science (JSPS;16P04150).

List of Symbols and SI Units

C_p	: Specific heat	J kg^{-1}K^{-1}
D	: Diameter	m
H	: Latent heat	kJ kg^{-1}
h_p	: Thermal conductivity of gas	W m^{-1}K^{-1}
K	: Number of ligaments	-
k	: Effective thermal conductivity	W m^{-1}K^{-1}
l	: Length	m
P	: Pressure	Pa
R	: Radius of rotating disk	m
r	: Distance from centre	m
T	: Temperature	K
t	: Time	s
We	: Weber number	-
$\mathrm{Z_R}$	: Ohnesorge number	-
ω	: Rotating speed	rps
μ	: Dynamic viscosity	Pa s
ρ	: Density of liquid	Kg m^{-3}
σ_s	: Surface tension of liquid	Dyne m^{-1}
v	: Velocity	m s^{-1}
ε	: Emissivity	-
σ	: Boltzmann constant	W m^{-2}K^{-4}

Subscripts:

g : Gas
s : Solid
l : Liquid

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