Simplified observer design and simulation for leak detection in fluid pipelines

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Abstract

A pipeline simulator and a procedure for leak detection and isolation are presented. The leak detection system uses a linearized system description in the design of observers. The design is based on extracting observable subsystems from the pipeline description which is done through the use of a numerical algorithm. The observer design is improved by finding the structure of the subsystems, providing an easier way to update the operating point and friction parameters of the system. Leak detection and isolation is tested against the pipeline simulator and the need for a correct operating point in the observers is confirmed.

1 Introduction

Pipelines transporting fluids are used to a great extent in the world today. Examples are pipelines transporting oil, both subsea and across land, water aqueducts and more. The reliable operation of such pipelines is very important, especially as more fragile environments are put at risk, such as arctic areas.

Leaks in such pipelines can be detected and located using several approaches, ranging from visual inspection to advanced model-based methods. The latter approach uses mathematical models of the pipeline based on fluid dynamics together with fault detection and isolation (FDI) theory to treat the measurements available in order to detect and locate leaks.

C. Verde has in [1] developed a method for leak detection in fluid pipelines using a linearized description of the system. The procedure is based on using a bank of observers where each observer is sensitive to all leaks but one. Together, the residuals from these observers provide the information necessary to detect and isolate multiple leaks in a pipeline. In this method, a numerical algorithm is used to extract observable subsystems needed for the FDI observer design. Due to the use of a linearized description, these subsystems would have to be generated numerically for every change in operating point, as well as when updating the friction parameter.

This paper presents a simpler way to generate the FDI observers for Verde's leak detection method as well as a pipeline simulator. The structure of the observable subsystems used to design the bank of FDI observers is given, where the operating point and friction parameters are easily available. This means it is no longer necessary to go through the numerical algorithm in order to update these parameters.

In Section 2, a pipeline simulator model and numerical scheme are presented. Then, in Section 3, the linearized pipeline description from [1] is introduced, followed by an overview of the leak detection and isolation procedure from [1] in Section 4. In Section 5, the simplified observer design is given, and after that, simulation results showing detection and isolation of a single leak is provided in Section 6. Finally, conclusions are given in Section 7.

2 Pipeline simulator

For simulating leaks in fluid pipelines, the author has continued development of a pipeline simulator started by Nilssen in [3]. The simulator consists of a nonlinear pipeline model and a known numerical scheme and will be used as a "real world" representation in the simulations. A simpler model will be used in observers for leak detection.

Using the principles of conservation of mass, momentum and energy, a mathematical model consisting of 1st order partial differential equations based on isothermal flow including temperature/viscosity dynamics have been developed. Because the flow is single-phase fluid flow, assumptions on low compressibility are assumed to hold.

The complete model is

$$\begin{aligned} \frac{\partial p}{\partial t} + u\frac{\partial p}{\partial z} + \rho c^2 \frac{\partial u}{\partial z} &= -\frac{c^2}{A} f_l(z) \\ \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial z} &= -\frac{f}{2}\frac{|u|u}{D} - gsin\beta + \frac{1}{A}\frac{u}{\rho} f_l(z) \\ \rho &= \rho_{ref} + \frac{p - p_{ref}}{c^2} \\ T(z)_u &= (T_{in} - T_g)e^{-\frac{K_1}{u}z} \\ \frac{\gamma}{\gamma_{20^\circ C}} &\approx exp\left[C\left(\frac{293}{T} - 1\right)\right], \end{aligned} \tag{1}$$

where t is the time, z the spatial variable, u the flow, p the pressure, ρ the density, c the speed of sound, A the cross-sectional area, D the diameter, β the inclination angle, f the friction coefficient, p_{ref} the reference pressure, ρ_{ref} the reference density, T_{in} the temperature at the inlet, T_g the surrounding temperature, γ the viscosity and $\gamma_{20^\circ C}$ the viscosity at room temperature. The last expression in (1) is used to determine the viscosity at a varying temperature, T, where C is a viscosity parameter.

A leak is modeled as a continuous function $f_l(z)$ of the position and describes mass per second per meter. For a point leak, $f_l(z)$ is the Dirac distribution.

It is assumed that temperature changes happen slowly, such that it only affects the viscosity. The principle for the inclusion of temperature dynamics in the model is to use a simplified profile for the pipeline, where the temperature is modeled as a function $T(z)_u$ of the spatial variable z. This way, the temperature can be updated at a slower rate than the rest of the system, leading to an updated viscosity which again leads to an updated friction parameter.

The friction coefficient is determined from

$$\frac{1}{\sqrt{f}} = -1.8log_{10}\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{Re_d}\right], \tag{2}$$

for turbulent flow and

$$f = \frac{64}{Re_d} \tag{3}$$

for laminar flow, where the Reynolds number is given by $Re_d = \frac{\rho u D}{\gamma}$ and ε is the pipeline roughness.

The developed model is of conservative type and can be written on a quasi-linear form

$$\frac{\partial \mathbf{U}}{\partial t} + \mathbf{A}(\mathbf{U})\frac{\partial \mathbf{U}}{\partial z} = \mathbf{S}(\mathbf{U}, z), \tag{4}$$

with

$$\begin{aligned} \mathbf{U} &= \begin{bmatrix} p \\ u \end{bmatrix} \\ \mathbf{A}(\mathbf{U}) &= \begin{bmatrix} u & \rho c^2 \\ \frac{1}{\rho} & u \end{bmatrix} \\ \mathbf{S}(\mathbf{U}, z) &= \begin{bmatrix} -\frac{1}{A} f_l(z) \\ -\frac{f}{2}\frac{u|u|}{D} - gsin\beta - \frac{1}{A} u f_l(z) \end{bmatrix}. \end{aligned} \tag{5}$$

This system is on a primitive variable form, which means it is expressed using physical variables. From this, the system can further be shown to be of type strictly hyperbolic by looking at the eigenvalues and eigenvectors of $\mathbf{A}(\mathbf{U})$.

In order to find a solution to the PDE including source, a method of splitting the problem into two subproblems is proposed. The idea is to combine a solution to the PDE without source with a solution to an ODE, using the solution from the PDE as initial conditions. The two subproblems can be presented like the following

$$\begin{array}{lcc} PDE & : & \frac{\partial \mathbf{U}}{\partial t} + \mathbf{A}(\mathbf{U})\frac{\partial \mathbf{U}}{\partial z} = 0 \\ IC & : & \mathbf{U}(z, t^n) = \mathbf{U}^n \\ ODE & : & \frac{d}{dt}\mathbf{U} = \mathbf{S}(\mathbf{U}) \\ IC & : & \bar{\mathbf{U}}^{n+1} \end{array}. \tag{6}$$

Splitting the problem like this provides the ability to select the best method for each subproblem independently.

For the solution to the PDE a MUSCL-Hancock scheme is selected (see [5]). The method is for systems on a primitive variable form and has 2nd order accuracy. The scheme is on the form

$$\mathbf{U}_i^{n+1} = \mathbf{U}_i^n + \frac{\Delta t}{\Delta z}\bar{\mathbf{A}}_i[\mathbf{U}_{i-\frac{1}{2}}^{n+\frac{1}{2}} - \mathbf{U}_{i+\frac{1}{2}}^{n+\frac{1}{2}}], \tag{7}$$

which is of type two-step, finite-difference on staggered grid. The idea is to find values for $\mathbf{U}_{i+\frac{1}{2}}^{n+\frac{1}{2}}$ and $\bar{\mathbf{A}}_i$, which then can be used to find the solution of (7).

Start by calculating the temporal step-size from

$$\Delta t = \frac{C_{cfl}\Delta z}{S^n_{max}}, \qquad (8)$$

where C_{cfl} is the Courant number and S^n_{max} is the maximum wave velocity over all segments at time t^n.

Next, extrapolate the boundary values at node i from

$$\begin{aligned} \mathbf{U}^L_i &= \mathbf{U}^n_i - \frac{1}{2}\Delta_i \\ \mathbf{U}^R_i &= \mathbf{U}^n_i + \frac{1}{2}\Delta_i, \end{aligned} \qquad (9)$$

where L indicates the value between node $i-1$ and i and R the value between node i and $i+1$ and

$$\Delta_i = \frac{1}{2}(1-\omega)\Delta_{i-\frac{1}{2}} + \frac{1}{2}(1+\omega)\Delta_{i+\frac{1}{2}} \qquad (10)$$

is a *slope curve* and

$$\begin{aligned} \Delta_{i-\frac{1}{2}} &\equiv \mathbf{U}^n_i - \mathbf{U}^n_{i-1} \\ \Delta_{i+\frac{1}{2}} &= \mathbf{U}^n_{i+1} - \mathbf{U}^n_i, \end{aligned} \qquad (11)$$

where $\omega \in [-1,1]$.

Then, evolve the boundary values using Δt according to

$$\bar{\mathbf{U}}^{L,R}_i = \mathbf{U}^{L,R}_i + \frac{1}{2}\frac{\Delta t}{\Delta z}\mathbf{A}(\mathbf{U}^n_i)(\mathbf{U}^L_i - \mathbf{U}^R_i), \qquad (12)$$

followed by finding $\mathbf{U}^{n+\frac{1}{2}}_{i+\frac{1}{2}}$ by solving the Riemann problem (see [5])

The only thing missing in order to use (12) to find $\mathbf{U}^{n+1}_i$ is $\bar{\mathbf{A}}_i$, which can be calculated from

$$\bar{\mathbf{A}}_i = \mathbf{A}(\bar{\mathbf{U}}_i), \qquad (13)$$

where $\bar{\mathbf{U}}_i = \frac{1}{2}(\bar{\mathbf{U}}^L_i + \bar{\mathbf{U}}^R_i)$. This concludes the solution to the PDE subproblem.

The set of ODEs to be solved are

$$\begin{aligned} \frac{d}{dt}p &= -\frac{1}{A}c^2 f_l(z), \\ \frac{d}{dt}u &= -\frac{f}{2}\frac{|u|u}{D} - gsin\beta - \frac{1}{A}c^2 u f_l(z). \end{aligned} \qquad (14)$$

These can be solved using a suitable ODE-solver in Matlab.

The boundary conditions, in the form of flows and pressures at the pipeline extremes, are implemented using interpolation. One of the flow/pressure values at each end is given as input to the system, while the other two values are obtained using interpolation of the values in neighboring segments.

3 Pipeline description for leak detection

For the leak detection and isolation procedure, a simpler pipeline description than the one used in the pipeline simulator will be used. The reason for this is that a simpler model provides more tools to develop such procedures. Also, the leak detection and isolation method does not need to provide information about all the states and parameters in the pipeline, as all that is interesting in this regard is information about faults in the form of leaks.

Based on the assumptions that the density and cross-sectional area of the pipeline are constant and that convective changes are negligible (the model for the pipeline simulator in (1) only assumes low compressibility), the dynamic and continuity equations for one-dimensional unsteady flow through closed conduits are (see [4], [1])

$$\begin{aligned} \frac{\partial Q}{\partial t} + gA\frac{\partial H}{\partial z} &= -\mu Q|Q|, & (15) \\ c^2\frac{\partial Q}{\partial z} + gA\frac{\partial H}{\partial t} &= 0, & (16) \end{aligned}$$

where Q is the volume flow, H the piezometric head, z the spatial coordinate, g the gravity, A the cross-sectional area and c the speed of sound. The friction parameter is given by $\mu = \frac{f}{2DA}$, where D is the pipeline diameter and f is the friction coefficient.

A leak can be modeled as a boundary condition between two pipeline sections as

$$Q^b|_{z_f} = Q^a|_{z_f} + Q|_{z_f}, \qquad (17)$$

where z_f is the leak position, Q^b and Q^a are the flows before and after the leak and $Q|_{z_f} = \lambda\sqrt{H|_{z_f}}$ is the leak flow with $\lambda \geq 0$.

By assuming n sections and $l = n-1$ leaks, the pipeline can be modeled using n pair of equations of the form (15)-(16) with a boundary condition in between. Using a finite difference discretization on a uniform grid for the spatial coordinate leads to n sets of nonlinear coupled equations on the form

$$\begin{aligned} \dot{Q}_i &= a_1(H_i - H_{i+1}) - \mu Q_i|Q_i| \quad \forall i = 1,\ldots,n, & (18) \\ \dot{H}_i &= a_2(Q_{i-1} - Q_i - \lambda_{i-1}\sqrt{H_i}) \quad \forall i = 2,\ldots,n, & (19) \end{aligned}$$

where $a_1 = \frac{gA}{\Delta z}$, $a_2 = \frac{c^2}{\Delta z g A}$, λ_i is the leak parameter and $\Delta z = \frac{L}{n}$ is the section length with L as the pipeline length. The boundary conditions are given by using the piezometric heads at the pipeline inlet and outlet, H_1 and H_{n+1}, as inputs.

Using a Taylor's series expansion, the system can be linearized around an operating point (x_0, u_0) and then written on a compact form as

$$\dot{x} = A_0 x + B_0 u + F\bar{\lambda} + \Delta o^2(x_0, u_0), \tag{20}$$

where the state, input and leak vectors are given by

$$x = \begin{bmatrix} Q_1 & H_2 & Q_2 & \cdots & Q_n \end{bmatrix}^T \in \Re^{2n-1}, \tag{21}$$
$$u = \begin{bmatrix} H_1 & H_{n+1} \end{bmatrix}^T \in \Re^2, \tag{22}$$
$$\bar{\lambda} = \begin{bmatrix} \bar{\lambda}_1 & \bar{\lambda}_2 & \cdots & \bar{\lambda}_l \end{bmatrix}^T \in \Re^l, \tag{23}$$

with $\bar{\lambda}_i := \lambda_i \left(\sqrt{x_{2i}|_{x_0}}\right)^{-1}$ as the linearized leak terms, $\lambda_i \geq 0$ and the uncertainty vector $\Delta o^2(x_0, u_0)$ is associated to all the higher order terms of the Taylor's series expansion. The system matrices are

$$A_0 = \begin{bmatrix} k & -a_1 & 0 & \cdots & 0 & 0 \\ a_2 & 0 & -a_2 & \cdots & 0 & 0 \\ 0 & a_1 & k & \ddots & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & \ddots & 0 & -a_2 \\ 0 & 0 & 0 & \cdots & a_1 & k \end{bmatrix} \tag{24}$$

$$B_0 = \begin{bmatrix} a_1 & 0 \\ \vdots & \vdots \\ 0 & -a_1 \end{bmatrix}, \tag{25}$$

$$F = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ -a_2 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ 0 & -a_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -a_2 \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \tag{26}$$

where $k = -2\mu q_0$ are the linearized friction effects. The flow q_0 in the operating point is constant, due to the assumption of constant density in the model.

The output equation is given by the measured flows at the inlet and outlet (Q_1 and Q_n) and can be written

$$y = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix} x = Cx. \tag{27}$$

The system (20) is observable using only one component of the output vector (i.e. one of the measured flows). This redundancy is used to solve the FDI problem.

4 Leak detection

The leak detection scheme developed by Verde in [1] is based on separating the leaks into two sets of faults, one set containing the faults of no interest in the form of a single leak location, and one set containing the rest of the leaks as the faults of interest. In order to detect and isolate any leak, a bank of observers is designed, where each observer is sensitive to a different set of faults. Evaluating all the residuals together provides the necessary information to detect and locate multiple leaks in the pipeline.

The leak vector $\bar{\lambda}$ can be separated into

$$\bar{\Lambda}_i = [\ \bar{\lambda}_1 \ \cdots \ \bar{\lambda}_{i-1} \ \ \bar{\lambda}_{i+1} \ \cdots \ \bar{\lambda}_l]^T \in \Re^{l-1} \tag{28}$$

and $\bar{\lambda}_i$, where $i = 1, ..., l$. This results in l system descriptions of the form

$$\dot{x}_i = A_0 x_i + B_0 u + K_i \bar{\Lambda}_i + E_i \bar{\lambda}_i + \Delta o^2(x_0, u_0) \ \ i = 1, ..., l, \tag{29}$$

where

$$K_i = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ -a_2 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ 0 & -a_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -a_2 \\ 0 & 0 & \cdots & 0 \end{bmatrix} \in \Re^{(2n-1)\times(l-1)} \tag{30}$$

$$E_i = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ -a_2 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \in \Re^{2n-1}. \tag{31}$$

For this system it can be shown (see [1]) that there exists a transformation $\overline{x_i} = T_i x$ that generates a system of the form

$$\begin{bmatrix} \dot{\overline{x}}_{i1} \\ \dot{\overline{x}}_{i2} \end{bmatrix} = \begin{bmatrix} \overline{A}_{i11} & \overline{A}_{i12} \\ \overline{A}_{i21} & \overline{A}_{i22} \end{bmatrix} \begin{bmatrix} \overline{x}_{i1} \\ \overline{x}_{i2} \end{bmatrix} + \begin{bmatrix} \overline{B}_{i1} \\ \overline{B}_{i2} \end{bmatrix} u + \begin{bmatrix} 0 \\ \overline{E}_i \end{bmatrix} \overline{\lambda}_i + \begin{bmatrix} \overline{K}_{i1} \\ \overline{K}_{i2} \end{bmatrix} \overline{\Lambda}_i + \overline{\Delta}, \quad (32)$$

$$y = \begin{bmatrix} \overline{C}_{i1} & \overline{C}_{i2} \end{bmatrix} \begin{bmatrix} \overline{x}_{i1} \\ \overline{x}_{i2} \end{bmatrix}, \quad (33)$$

where the $\overline{x_{i1}}$ subsystem is insensitive to $\overline{\lambda}_i$ and $\overline{\Delta}$ is the uncertainty.

To design the observers that generate residuals for the $\overline{x_{i1}}$ subsystems in (32), Verde proposes the use of an algorithm developed by Hou and Müller in [2]. The idea behind this algorithm is to use an unknown-input observer design to develop FDI observers where the faults of no interest are treated as unknown inputs. This results in an observable subsystem where only the effects of the faults of interest show up, which can then be used to design an FDI observer for the original system. The algorithm is easy to implement numerically for the pipeline leak detection case.

The application of the Hou and Müller algorithm on the system in (29), results in an observable subsystem of the form

$$\begin{aligned} \dot{\xi}_i &= \tilde{A}_i \xi_i + \tilde{G}_i y a_i + \tilde{B}_i u + \tilde{M}_i \overline{\Lambda}_i + \wp, \\ z_i &= \tilde{C}_i \xi_i = T_{1i} y, \end{aligned} \quad (34)$$

where ya_i is an auxiliary output and $\wp$ is a disturbance independent on $\overline{\lambda}_i$. The subsystem can be obtained for any number of leaks.

From the subsystem in (34) an observer is designed for each fault set $\overline{\Lambda}_i$ as

$$\begin{aligned} \dot{\hat{x}}_i &= \tilde{A}_i \hat{x}_i + \tilde{G}_i y a_i + \tilde{B}_i u + K_{fi}(z_i - \hat{z}_i), \\ \hat{z}_i &= \tilde{C}_i \hat{x}_i, \end{aligned} \quad (35)$$

where $\hat{x}_i$ is the estimation vector, $\hat{z}_i$ the output estimation and K_{fi} the observer gain. The residual is given by

$$r_i(t) = (z_i - \hat{z}_i) = (T_{1i} y - \hat{z}_i) \quad (36)$$

and is zero in response to $\overline{\lambda}_i \geq 0$ and nonzero in responce to $\overline{\Lambda}_i \neq 0$. A Kalman filter is proposed for the observer gain calculation.

In order to detect and isolate leaks in the pipeline, all the residuals have to be evaluated. To do this, a set of indicator functions are proposed in [1]. For constant holes vector λ, the following relationship between residuals and holes hold

$$r_\infty = \lim_{t \to \infty} \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_l \end{bmatrix} = R_{ss} \lambda, \quad (37)$$

where R_{ss} is a non-singular matrix containing the static gains from the leaks to the residuals for the linearized model (see [1]). The faults can then be estimated by

$$\hat{\lambda} = (R_{ss})^{-1} r_\infty. \quad (38)$$

The normalized indicator functions are proposed as

$$J_i = \frac{\hat{\lambda}_i}{||\hat{\lambda}||_2} \quad for\ i = 1, \ldots, l, \quad (39)$$

where a value of J_i above a positive threshold indicates a leak in section i.

5 Simplified observer design

A novel result in this paper is the discovery that the observable subsystem in (34) for the linearized pipeline system (29) can be generated directly without using the numerical Hou and Müller algorithm.

By examining the use of the Hou and Müller algorithm on the system in detail, the effect each step of the algorithm have on the system can be seen in terms of the equations in (18)-(19). The particular structure of the linearized pipeline description makes it possible to write the form of subsystems equivalent to those resulting from the algorithm directly.

There are three different cases to consider, namely if the leak position is to the left of the middle ($i \leq \frac{l}{2}$), to the right of the middle ($i \geq \frac{l}{2} + 1$) or (for odd number of leaks only) exactly in the middle ($i = \frac{l+1}{2}$). The reason for this is the difference in the use of the output measurements as auxiliary output.

For all three cases, the $\tilde{A}_i$ matrix has the following structure

$$\tilde{A}_i = \begin{bmatrix} k & -a_1 & 0 & \cdots & 0 & 0 \\ a_2 & 0 & -a_2 & \cdots & 0 & 0 \\ 0 & a_1 & k & \ddots & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & \ddots & 0 & -a_2 \\ 0 & 0 & 0 & \cdots & a_1 & k \end{bmatrix}, \quad (40)$$

where

$$\tilde{A}_i \in \Re^{(2(n-i)-1)\times(2(n-i)-1)} \; if \; i \leq \frac{l}{2}, \quad (41)$$

$$\tilde{A}_i \in \Re^{(2i-1)\times(2i-1)} \; if \; i \geq \frac{l}{2}+1, \quad (42)$$

and

$$\tilde{A}_i \in \Re^{(n-1)\times(n-1)} \; if \; i = \frac{l+1}{2} \quad (43)$$

and the number of leaks is odd. The matrix can also be written as

$$\tilde{A}_i = \bar{A}_i + diag(k, 0, k, ..., k) = \bar{A}_i + \bar{K}_i, \quad (44)$$

where the parts affected by the linearization and friction parameter μ have been separated out. This means $\bar{A}_i$ is a constant matrix and only have to be generated once.

The new state vector, ξ_i, gives the information needed to see how the new subsystem was created and is given by

$$\xi_i = \begin{bmatrix} x_{2i-1} + x_{2i+1} \\ -(x_{2i-2} - x_{2i+2}) \\ \vdots \\ x_1 + x_{4i-1} \\ x_{4i} \\ \vdots \\ x_{2n-1} \end{bmatrix} \in \Re^{2(n-i)-1}, \quad (45)$$

if $i \leq \frac{l}{2}$,

$$\xi_i = \begin{bmatrix} x_1 \\ \vdots \\ x_{4i-2n-2} \\ x_{4i-2n-1} + x_{2n-1} \\ \vdots \\ x_{2i-2} - x_{2i+2} \\ x_{2i-1} + x_{2i+1} \end{bmatrix} \in \Re^{2i-1}, \quad (46)$$

if $i \geq \frac{l}{2}+1$ and

$$\xi_i = \begin{bmatrix} x_{2i-1} + x_{2i+1} \\ -(x_{2i-2} - x_{2i+2}) \\ \vdots \\ x_1 + x_{2n-1} \end{bmatrix} \in \Re^{n-1}, \quad (47)$$

if $i = \frac{l+1}{2}$ (odd number of leaks).

It is worth noting that when the leak is to the right of the middle, the states containing a difference between piezometric heads have the opposite sign compared to when the leak is to the left of the middle. This is done in order to keep the same structure for the $\tilde{A}_i$ matrix.

The other matrices are

$$\tilde{B}_i = \begin{bmatrix} 0 & 0 \\ \vdots & \vdots \\ a_1 & 0 \\ \vdots & \vdots \\ 0 & -a_1 \end{bmatrix} \Leftarrow row \; 2i-1, \quad (48)$$

$$\tilde{G}_i = \begin{bmatrix} 0 & 0 \\ \vdots & \vdots \\ -a_2 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix} \Leftarrow row \; 2i, \quad (49)$$

$$\tilde{C}_i = \begin{bmatrix} 0 & \cdots & 0 & 1 \end{bmatrix} \in \Re^{2(n-i)-1}, \quad (50)$$

if $i \leq \frac{l}{2}$,

$$\tilde{B}_i = \begin{bmatrix} a_1 & 0 \\ \vdots & \vdots \\ 0 & -a_1 \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix} \Leftarrow row \; 4i-2n+1, \quad (51)$$

$$\tilde{G}_i = \begin{bmatrix} 0 & 0 \\ \vdots & \vdots \\ 0 & a_2 \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix} \Leftarrow row \; 4i-2n, \quad (52)$$

$$\tilde{C}_i = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} \in \Re^{2i-1} \quad (53)$$

if $i \geq \frac{l}{2}+1$ and

$$\tilde{B}_i = \begin{bmatrix} 0 & 0 \\ \vdots & \vdots \\ a_1 & -a_1 \end{bmatrix}, \quad (54)$$

$$\tilde{C}_i = \left[\begin{array}{cccc} 0 & \cdots & 0 & \frac{1}{\sqrt{2}} \end{array} \right] \in \Re^{n-1} \qquad (55)$$

if $i = \frac{l+1}{2}$ (odd number of leaks).

There is no need for an auxiliary output ($\tilde{G}_i = 0$) when the leak is in the middle of the pipeline, as both flow measurements will be used as ordinary outputs. The $\tilde{B}_i$, $\tilde{G}_i$ and $\tilde{C}_i$ matrices are constant and only need to be generated once for each leak.

The faults of interest are contained in $\tilde{M}_i\overline{\Lambda_i}$ and there is no need to explicitly define the details of this term, as it will not be used in the FDI observer. The important thing is that all the faults of interest are still part of the system. This means that by removing this term from the system when designing an observer, any such fault would generate an error in that observer.

An advantage of being able to write the subsystem matrices directly, is the fact that the parameter k in $\overline{K_i}$ from (44) contains the parts affected by the linearization and the friction parameter in the observer design. The friction and operating point are the only parameters that change in the linearized pipeline description during operation, which means that all parameters that need to be adjusted are available in one parameter in the subsystem definition and there is no need to go through a numerical algorithm to update them.

6 Simulation results

Simulations are performed on an 8000m pipeline with a diameter of four inches ($D = 0.1016m$). The adimensional friction coefficient used is $f = 0.0075$, the bulk constant is $K = 1.31 \times 10^9$ and the reference pressure and viscosity are given by $p_{ref} = 101325$Pa and $\gamma_{ref} = 1.04 \times 10^{-1}$. The pipeline is simulated with 16 sections. At time $t = 40s$, a leak of size approximately 5% occurs at 3000m from the inlet of the pipeline. This is seen in Figure 1 and 2.

The leak detection uses 8 sections, which means there are 7 defined leak positions and thus 7 residuals to be generated. The development of the residuals can be seen in Figure 3. In the leak detection principle used, a leak is closer to the residuals with the lowest value and farther away from the ones with a higher value. In this case, the residual responses indicate that the leak is somewhere in section 3 close to the boundary between section 3 and 4. This corresponds well with

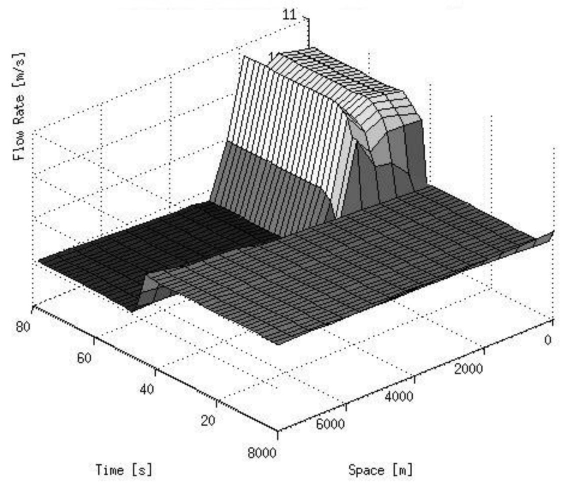

Figure 1: Flow from the pipeline simulation.

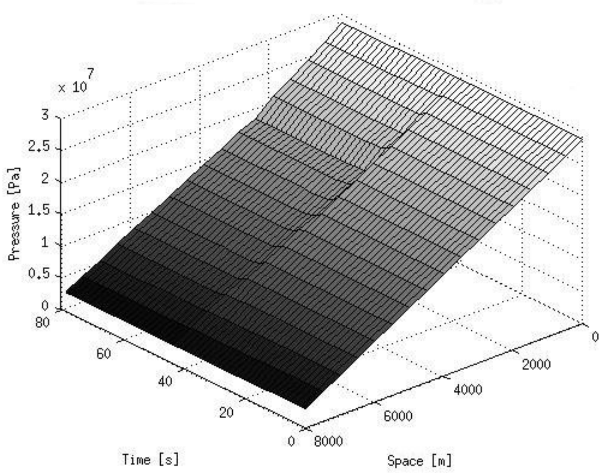

Figure 2: Pressure from the pipeline simulation.

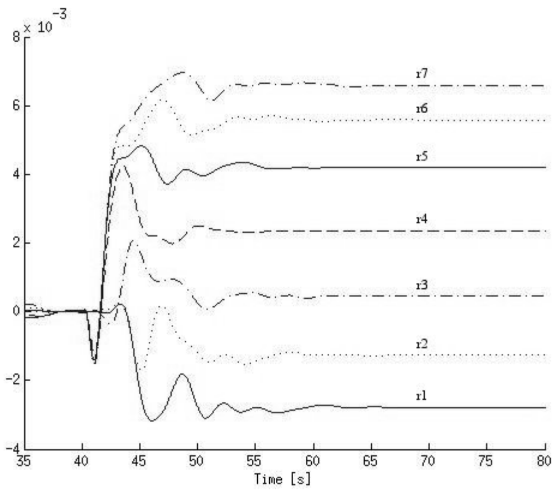

Figure 3: Residual development for the leak detection.

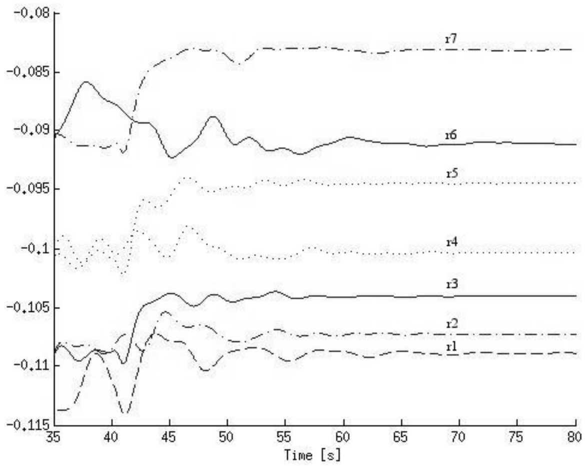

Figure 4: Residual development for the leak detection using a faulty operating point.

the position of the leak used in the simulation.

In order to indicate how much a change in operating point affects the residuals, results using an operating point of $\frac{1}{2}$ times the correct one in the observers are shown in Figure 4. From this it can be concluded that when not using the correct operating point, the resulting model error has a significant impact on the residual generation. This makes it impossible to use the residuals to locate the leak using the procedure as presented.

7 Conclusions

In this paper, a pipeline simulator and a method for leak detection and isolation have been presented. The main contribution is in showing the structure of observable subsystems used to develop FDI observers for the leak detection procedure. This simplifies the process of updating the friction parameter and operating point for the observers, as subsystems would otherwise have to be generated by a numerical algorithm for every update. Simulations confirm the effectiveness of the procedure in detecting and locating a single leak using the pipeline simulator. They also show how using the wrong operating point affects the effectiveness of the procedure, verifying the advantage of knowing the structure of the observable subsystems.

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