

Stochastic modeling of Transonic Flutter

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Abstract

The phenomenon of large amplitude self sustained oscillations in an aeroplane wing is known as *flutter*. In this article we describe our effort to improve an already existing mathematical model of flutter. Our work falls into two categories. Making the model fit the experimental data, and investigating how additive noise affects the model, including how noise can make a subcritical Hopf bifurcation look supercritical. To this end we have developed some numerical tools among these a continuation routine and a fourth order Runge Kutta method capable of integrate systems with noise.

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1 Introduction

At The National Aerospace Laboratory (NAL), Japan, experiments have been conducted on a high aspect ratio aeroplane wing.¹ The experiments were performed in a wind tunnel at transonic velocities. The wing was mounted on the ceiling to minimize the effect of gravitation. Figure 1 shows a drawing of the wing indicating the position of the strain gages and accelerometers used to record the motion of the model in the wind tunnel. The velocity of the airflow around the wing and the pressure in the wind tunnel can be changed continuously, thereby changing the dynamical pressure on the wing surface. At low dynamical pressures, the noise in the wind tunnel excites the wing and small vibrations around a stable fixpoint is observed. As the dynamical pressure is increased a critical point is reached and the wing start to oscillate with a large amplitude at a frequency of about 22 Hz. These oscillations are known as flutter. In mathematics this is described as a

¹A long, slim wing as on most passenger planes, as compared to the delta wing of a typical jet fighter.

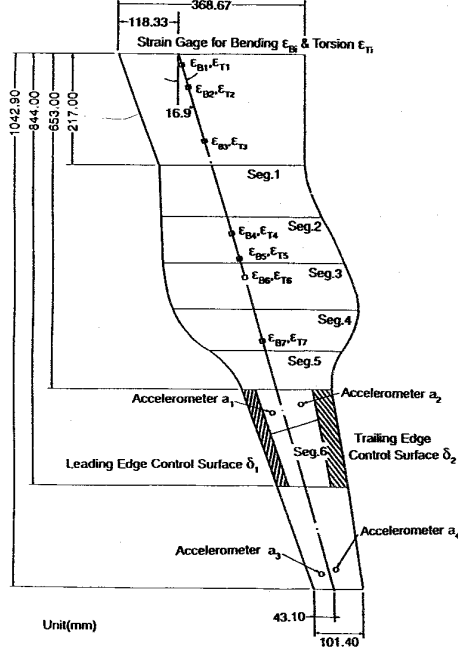


Figure 1: High aspect ratio wing model.

subcritical Hopf bifurcation connected to a saddlenode bifurcation, see section 3. However in a single experiment the wing seemed to undergo a supercritical bifurcation.

To get a better understanding of the flutter phenomenon a mathematical model has been developed. The model consists of a two parts, a linear part built from aerodynamic and aeroelastic theory and a nonlinear part added with little concern to the physical reality. The linear part describes the torsional and bending modes of the wing, in the simplest form only the first bending and torsional modes are taken into account yielding a two degrees of freedom model, in a more complex version six degrees of freedom are used. The nonlinear part of the model is responsible for the creation of a subcritical Hopf bifurcation.

As noise is present in the experiments we have examined the effects of adding noise while simulating the model. In order to do

this we have developed and tested numerical tools to integrate the system. The routine used is a classical fourth order Runge-Kutta enhanced with noise terms.

The temporal behavior produced by the integrator reproduces the experimental observed behavior very well. In addition to reproducing the usually observed flutter scenario it has also been possible to reproduce a temporal behavior that looks like a supercritical Hopf bifurcation as has been observed in one experiment.

2 Integrating noise

In order to reproduce the noisy temporal behavior observed in the wind tunnel experiments it has been necessary to make an integration scheme that includes additive noise. An Euler integration scheme is often used for this purpose but in order to speed up the calculations we have implemented a modified classical fourth order Runge Kutta integration routine with fixed step size. To each of the four coefficient a noise term has been added. The routine can be written as:

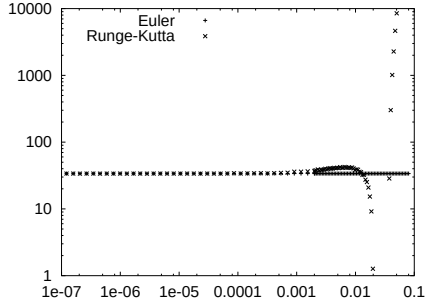
$$\begin{aligned}\tilde{k}_1 &= f(x_n, t_n)h + w_n\sqrt{\sigma^2h} \\ \tilde{k}_2 &= f(x_n + \tilde{k}_1/2, t_n + h/2)h + w_n\sqrt{\sigma^2h} \\ \tilde{k}_3 &= f(x_n + \tilde{k}_2/2, t_n + h/2)h + w_n\sqrt{\sigma^2h} \\ \tilde{k}_4 &= f(x_n + \tilde{k}_3, t_n + h)h + w_n\sqrt{\sigma^2h}\end{aligned}$$

The step then becomes:

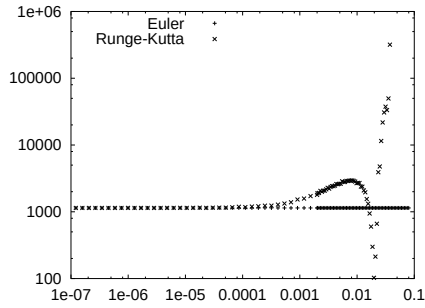
$$x_{n+1} = x_n + \frac{1}{6}(\tilde{k}_1 + 2\tilde{k}_2 + 2\tilde{k}_3 + \tilde{k}_4)$$

w_n is white noise and σ is the standard deviation of the noise. Note that the $\tilde{k}_n$'s includes a multiplication with the stepsize and that the noise term is multiplied by $\sqrt{h}$ this ensures convergence for the algorithm.

In order to check the performance of the integrator the simpler Euler has been implemented for comparison. The first two



(a) The first Kramers-Moyal coefficient



(b) The second Kramers-Moyal coefficient

Figure 2: Kramers-Moyal coefficients

Kramers-Moyal coefficients has been calculated using an average over 2000 steps and a large number of different stepsizes, see Figure 2(a) and 2(b). The Kramers-Moyal coefficients obtained using the Euler method are very good as they are almost independent of the stepsize whereas the fourth order Runge-Kutta needs steps shorter than $2 \cdot 10^{-4}$ in order to obtain the same property. Test runs with the two methods on our set of stochastic differential equations shows that the Runge-Kutta is stable for all stepsizes upto 10^{-4} whereas the Euler method fails to stay in the vicinity of a stable solution for stepsizes larger than 10^{-6} .

3 Bifurcations

The two different types of Hopf bifurcations are central in this work this section will give a brief introduction to the subcritical Hopf bifurcation and the supercritical Hopf bifurcation.

In a Hopf bifurcation a fixpoint (steady state solution) loses stability and limit cycle oscillations occur (a standing wave). In a supercritical Hopf bifurcation the amplitude of the oscillations grows slowly as seen in figure 3(a). In the subcritical bifurcation the solution suddenly jumps from the fixpoint to a large amplitude oscillation, figure 3(b).

The supercritical bifurcation is the simplest of the two, in this a stable fixpoint turns unstable and a stable limitcycle appear in the bifurcation point. The amplitude of this oscillation grows as the square root with the bifurcation parameter. In the subcritical bifurcation almost the same thing happens, a stable fixpoint turns unstable and a limit cycle appear. However in this bifurcation the limit cycle is unstable, this means that a separation of state space occur. In the most common case the unstable limit cycle again turns stable in a saddlenode bifurcation. The result of this is that when the Hopf bifurcation point is reached the solution jumps to the stable large amplitude limit cycle.

4 Modelling

The model we have been working with is a 6 dimensional set of differential equations with one nonlinearity. The nonlinearity has been implemented so that it is in correspondence with the present understanding of the underlying physics. The parameters in the

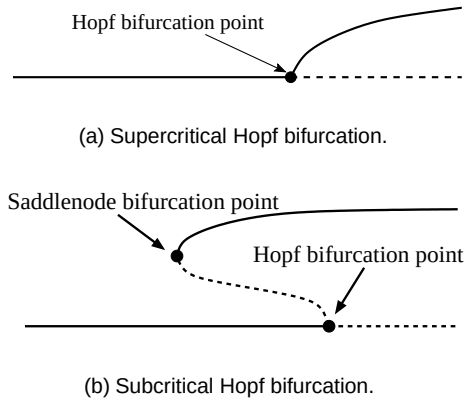


Figure 3: The two different types of Hopf bifurcations.

model are two generalized coordinates, q_i , the first derivatives and aerodynamic delays, Z_i . The amplitude of q_1 is referred to as the amplitude of an oscillation. The state vector is:

$$\mathbf{x} = (q_1, q_2, \dot{q}_1, \dot{q}_2, Z_1, Z_2)^\top$$

The model can be written as:

$$\dot{\mathbf{x}} = \mathbf{A}(v)\mathbf{x} + \mathbf{b}(\mathbf{x}, v)$$

where v is the velocity of the airflow. The $\mathbf{A}(v)$ matrix is determined using methods for subsonic flows. A factor has been multiplied to bring the numerically found bifurcation point in agreement with the experimentally found velocity. We have used a typical flutter case from the tests performed in May 1997 (F6-17) where the velocity was $v = 266$ m/s.

The nonlinear part $\mathbf{b}(\mathbf{x}, v)$ consists of second and fourth order terms in $\mathbf{x}$ and is added to the acceleration of the first bending mode. This is done to achieve the experimentally found sub-critical Hopf bifur-

cation. The vector is given as:

$$\mathbf{b}(\mathbf{x}, v) = \begin{bmatrix} 0 \\ 0 \\ a_{33}(v) (-\alpha q_1^2 + \beta q_1^4) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The two coefficients α and β can be used to move the saddlenode bifurcation point. Increasing α decreases the amplitude at the bifurcation point and lowers the corresponding value of the bifurcation parameter, whereas increasing β mainly decreases the amplitude. This can be used to select any location of the saddlenode bifurcation point.

This model has been used in most of the work. Additionally we have used a normal-form of a subcritical Hopf bifurcation with fourth order terms added to give a saddlenode bifurcation. The over all behavior of this is like the one sketched in Figure 3(b).

4.1 Temporal behaviour

The typical temporal behaviour found during the wind tunnel tests was as shown in figure 5(a). Noise excites the wing and small vibrations around the stable fixpoint are seen. The dynamical pressure is slowly increased until at a certain point the oscillations grow in amplitude. The large oscillations arising have a frequency of 22 Hz. After a short period the oscillations are stopped to avoid damaging the wing. This behaviour is typical for a system undergoing a subcritical Hopf bifurcation.

Several wind tunnel test have been performed to determine the exact dynamical pressure for the onset of the large oscillation. Figure 4 indicates how a noisy system

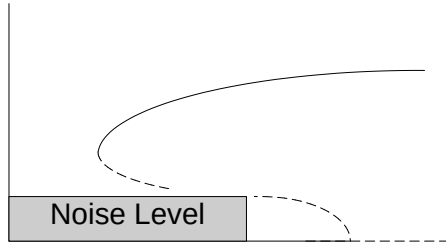


Figure 4: Noise creates small vibrations around the stable fixpoint. At some critical point the noise is sufficient to push the solution past the unstable limit cycle (separatrix) and large limitcycle oscillations occur. It is clear that the critical point may vary between experiments.

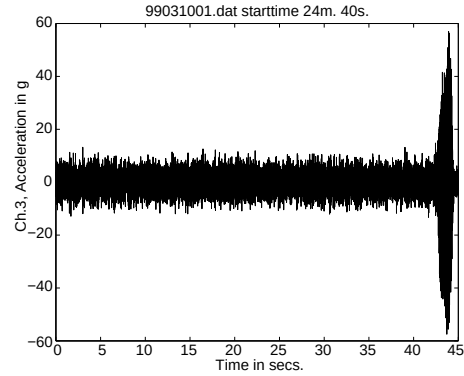
behaves near the Hopf bifurcation and also show why the critical point is not well defined.

In figure 5(b) the result of a numerical simulation can be seen. The mathematical model is clearly able to produce temporal behaviour close to what is seen in the experiments.

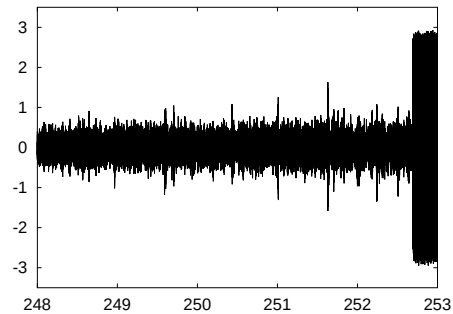
4.2 Supercritical Hopf bifurcation

In a single wind tunnel experiment the behaviour was different than what was normally found. Instead of the normal sudden jump to large amplitude oscillations, small oscillations was seen, and as the dynamic pressure was decreased the amplitude of these oscillations also decreased. As can be seen on figure 3(a) this is what would be expected if a supercritical Hopf bifurcation was present in the system. All efforts to find the location of this supercritical bifurcation experimentally has been unsuccessful.

Using a normal form of a subcritical Hopf bifurcation we have established that



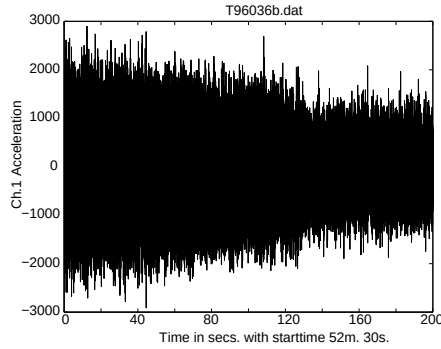
(a) Experimental behaviour. When large amplitude oscillation occur the experiment is stopped imidiatly to avoid damage of the wing.



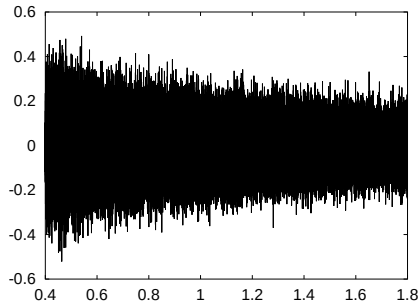
(b) Numerical simulation. The ordinate show the velocity and the abscissa the amplitude. The figure should only be compared qualitatively to figure 5(a).

Figure 5: Temporal behaviour. Noise creates small vibrations around the stable fixpoint. As the dynamic pressure is increased a critical point is reached and the amplitude of the oscillations grows.

it is possible to find a temporal behaviour similar to what would be expected if a supercritical Hopf bifurcation occurred in the system.



(a) Experimental behaviour.



(b) Numerical simulation. The ordinate show the bifurcation parameter and the abscissa the amplitude. As in figure 5 numerical behaviour should only be compared qualitatively to the experimental behaviour.

Figure 6: What appears to be the result of a supercritical Hopf bifurcation can be found by applying noise to a subcritical bifurcation.

When a low level noise is applied to the normal form of the subcritical bifurcation below the Hopf bifurcation point, what appears to be small amplitude limit cycle oscillations can be observed. When the bifurcation parameter is decreased the ampli-

tude of the oscillations also decreases. The experiments and the numerical simulation are qualitatively similar see figures 6(a) and 6(b).

The behavior of the system could be explained by the fact that at the Hopf bifurcation point two complex eigenvalues cross the imaginary axis and the solution becomes unstable. However, just before the bifurcation the real part of the two complex conjugated eigenvalues is negative and close to zero, hence the damping is small. Further away from the bifurcation, the real part of the eigenvalues becomes more and more negative, hence the damping is larger, and the disturbances introduced by the noise are attracted faster to the fixed point.

We have not been able to produce these results with our model of the wing. However given that the bifurcation phenomena are the same we now believe that a subcritical bifurcation is solely responsible for the wing dynamics.

5 Conclusion

An already existing mathematical model has been improved and analyzed. To get a better understanding of the flutter phenomena and how noise influences the wing, noise has been added to the system. This has been done by adjusting a fourth order Runge Kutta with fixed time step. The integrator has been tested and the Kramers-Moyal coefficients show that the method is stable for step sizes less than 10^{-4} .

The improvements of the model includes an analysis of the nonlinearities and the physics behind, and an examination of how the nonlinear coefficients can be changed to secure that the bifurcations are at the correct

locations.

Comparing the experimental results with the numerical results reveals good correspondence between the model and the experiments. The temporal behavior produced by numerical simulations are very similar to the behavior seen in the wing. What in the experiments appeared to be a supercritical Hopf bifurcation can be reproduced by a model which only contain a subcritical bifurcation.

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